Wrocław University of Technology

Obliczenia metodą dokładnej diagonalizacji hamiltonianu w kontekście cieczy elektronowych i kwantowego efektu Halla

Arkadiusz Wójs

Instytut Fizyki
Politechnika Wrocławska

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Outline

1. Quantum liquids
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3. Computations
4. Perspectives/applications
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1. Quantum liquids

Fractional quantum Hall effect

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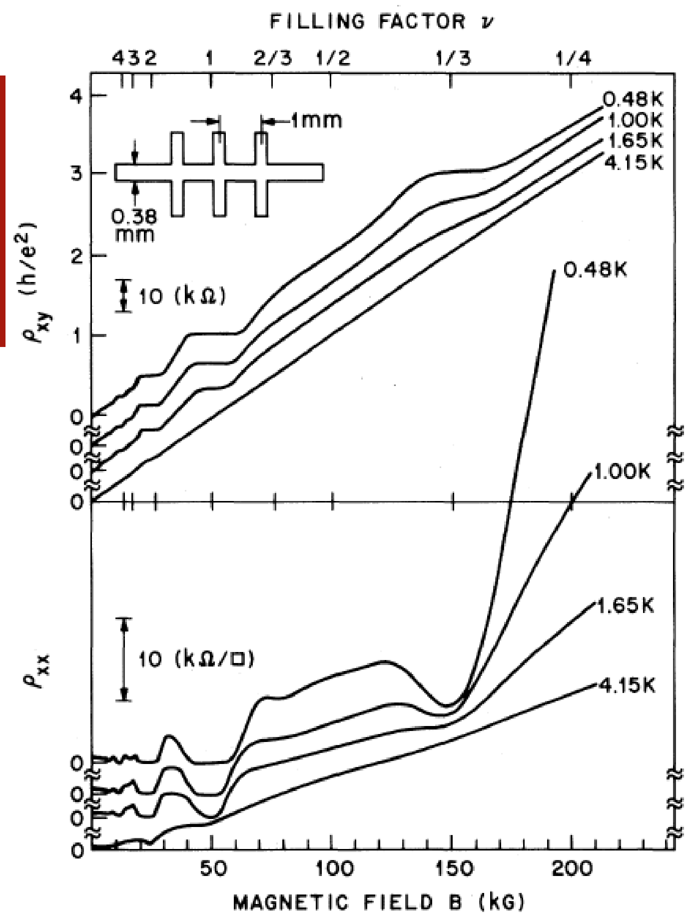
Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,[a],[b] H. L. Stormer,[a] and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 5 March 1982)

A quantized Hall plateau of $\rho_{xy} = 3h/e^2$, accompanied by a minimum in ρ_{xx}, was observed at $T < 5$ K in magnetotransport of high-mobility, two-dimensional electrons, when the lowest-energy, spin-polarized Landau level is $\frac{1}{3}$ filled. The formation of a Wigner solid or charge-density-wave state with triangular symmetry is suggested as a possible explanation.

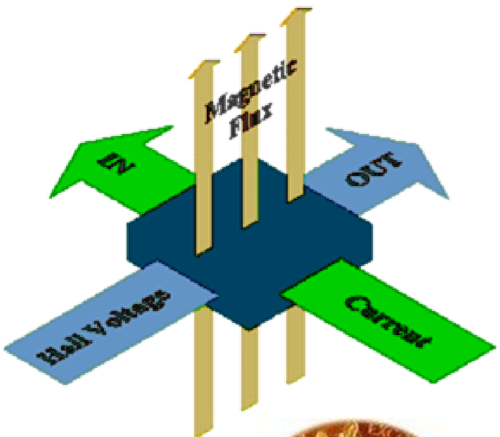
PACS numbers: 72.20.My, 71.45.-d, 73.40.Lq, 73.60.Fw



2D electron gas
high magnetic field
low temperature

→ Quantized Hall effect and vanishing longitudinal resistance

→ **electron quantum liquid**



$$\mathbf{E} = \sigma\,\mathbf{j} \qquad \sigma = R_K^{-1}\begin{bmatrix} 0 & \nu \\ -\nu & 0 \end{bmatrix}$$

$$\mathbf{j} = \rho\,\mathbf{E} \qquad \rho = R_K\begin{bmatrix} 0 & -\nu^{-1} \\ \nu^{-1} & 0 \end{bmatrix}$$

Klitzing constant : $R_K = h/e^2$

Many-electron states in lowest Landau level

Single-electron Hamiltonian

$$H = \frac{1}{2m}\left(p - \frac{e}{c}A\right)^2$$

Symmetric gauge

$$A = -\frac{B}{2}(y, x, 0)$$

Single-electron states (monomial × tail)

$$\varphi_m \propto z^m e^{-|z|^2/4\lambda^2}$$

(m=angular momentum)

Many-electron states (polynomial × tail)

$$\Psi = P(z_1, z_2, z_3, \ldots) \times \exp\left[-\sum_i \left|\frac{z_i}{2\lambda}\right|^2\right]$$

P is antisymmetric (fermions), and:

$$\deg(P) = N_\phi = N/\nu$$

Where N_ϕ is the highest allowed m
= the number of single-electron states
= Landau level degeneracy
= magnetic flux Φ=BA (in units ϕ_0=hc/e),

and ν=N/N_ϕ is the „filling factor"

(In higher Landau levels, Ψ has poles)

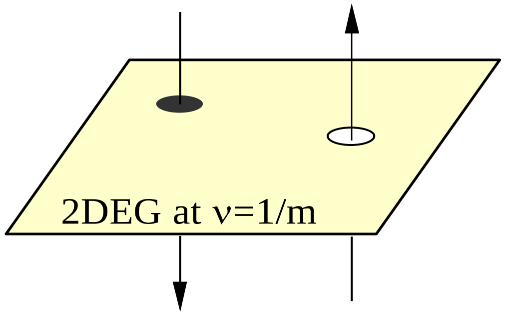
Laughlin ν=1/3 incompressible liquid

Full lowest Landau level (ν=1):

$$\Psi_1 = \det \begin{bmatrix} 1 & 1 & 1 & \dots \\ z_1 & z_2 & z_3 & \dots \\ z_1^2 & z_2^2 & z_3^2 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} e^{-\sum_i |z_i|^2/2\lambda^2}$$

$$= \prod_{i<j} (z_i - z_j) e^{\dots}$$

(Vandermonde)



$$c^+_{QH} \sim \prod_i z_i; \quad c^+_{QE} \sim \prod_i \partial/\partial z_i$$

Laughlin ν=1/3 wave function: $\Psi_{1/3} = \prod_{i<j} (z_i - z_j)^3 e^{\dots}$

Adiabatic injection of flux quantum through thin solenoid
→ new eigenstate, with ±e/3 charge accumulation/depletion

Quasiparticles with fractional charge (confirmed)
→ concept of anyon quantum statistics

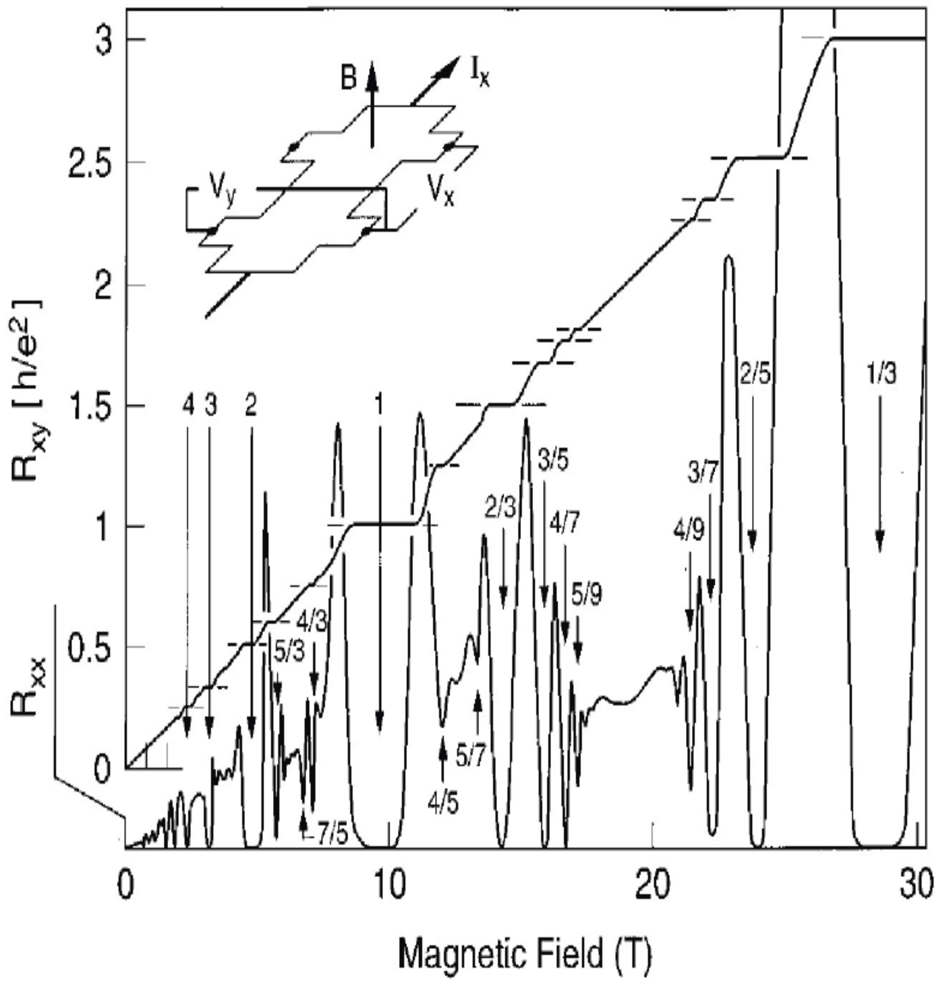
Problem: explain incompressibility

FQHE →
incompressible ground state
gapped quasiparticles, localization

Multiple fractions
(**universal**: material, size, shape, disorder, temperature, fields, …)

Wave functions: anti-symmetric polynomials of fixed degree N/ν

Need to explain this



2. Composite fermions

Jain's composite fermions

Filling factor = ν

N interacting electrons in lowest Landau level of degeneracy $N_\phi = N/\nu$

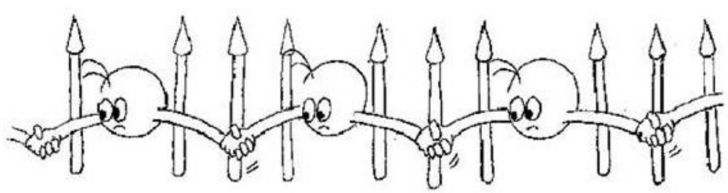
Ψ = anti-symmetric polynomial of degree N_ϕ

$\Psi_{z2\ldots zN}(z_1)$ has N_ϕ zeros (vortices):

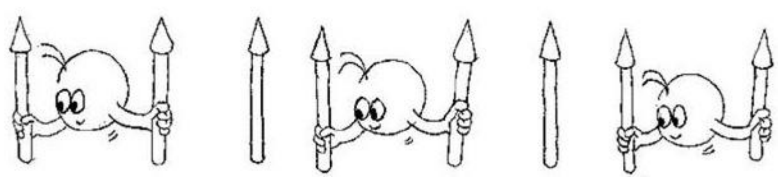
N-1 fixed at $z_2 \ldots z_N$ (Pauli exclusion principle); others mobile

Composite fermions: bound states of electrons and 2p vortices

Number of vortices = LL degeneracy = magnetic flux (in units of hc/e)



Electrons with strong (Coulomb) interaction in strong magnetic field B



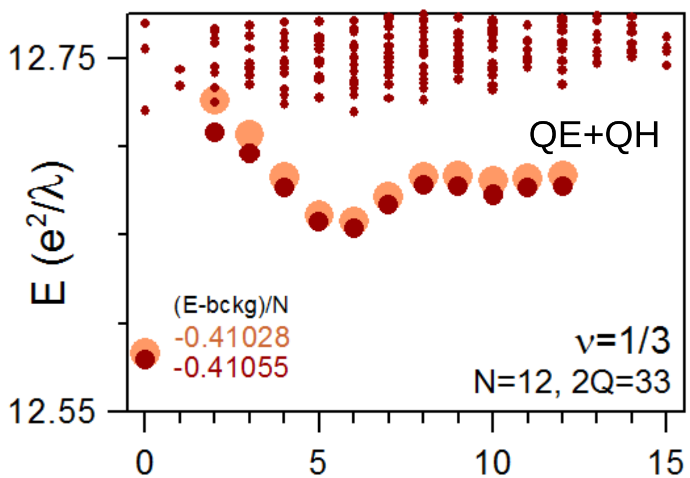
Composite fermions with weak residual interaction in weak magnetic field B*

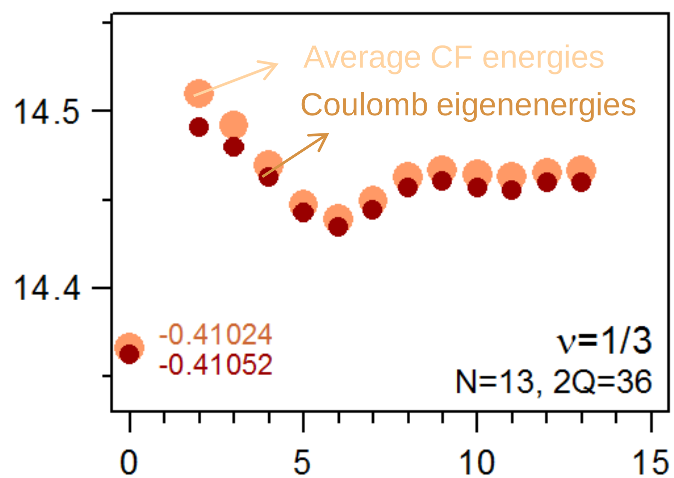
(Kwon Park)

$$\Phi^* = \Phi - N \cdot 2p \cdot hc/e; \quad B^* = B - (2p \cdot hc/e)\rho; \quad (\nu^*)^{-1} = \nu^{-1} - 2p$$

3. Computations

Evidence from numerics





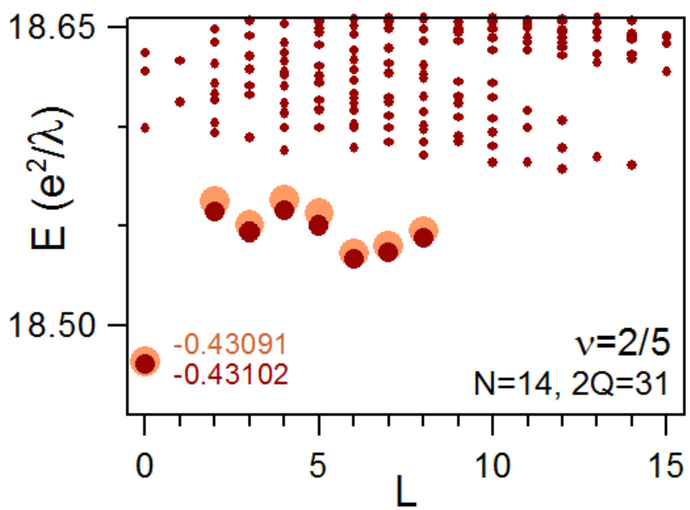
N electrons on sphere

radial magnetic field from a monopole 2Q

2Q=magnetic flux through surface

LL degeneracy=2Q+1

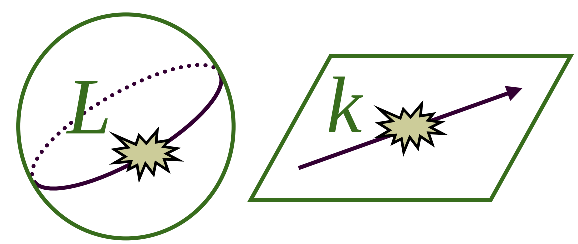
$\nu \sim N/2Q$



L = total angular momentum

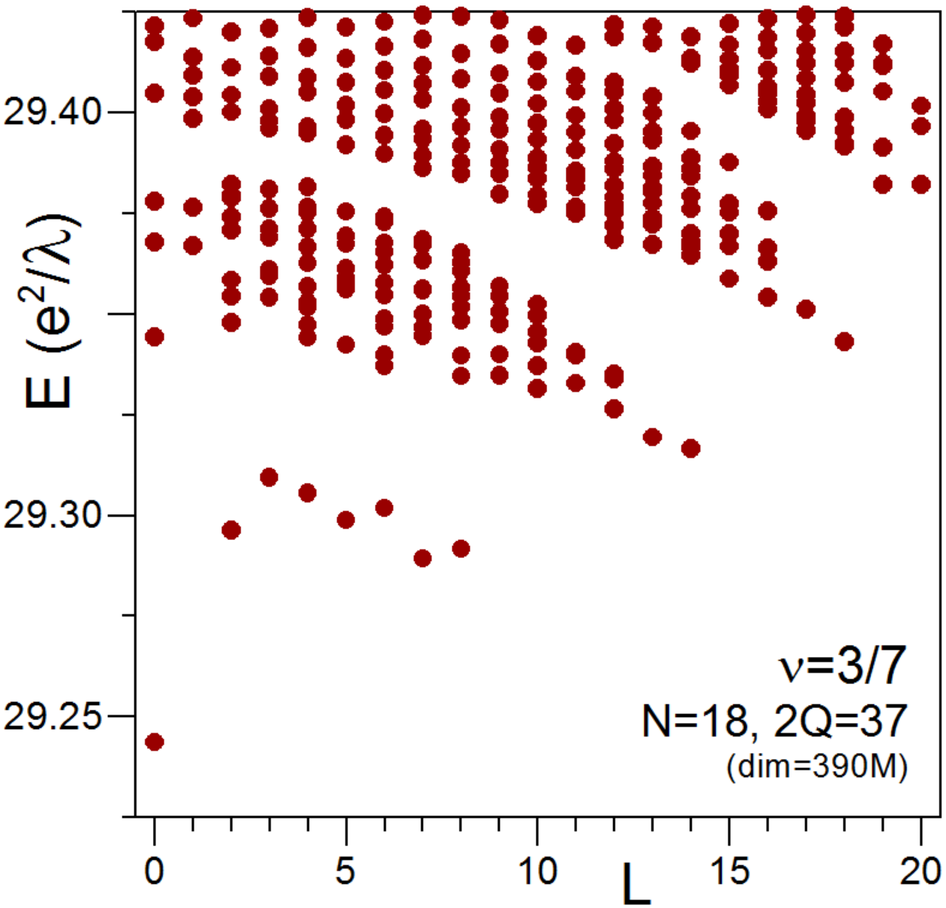
E = total Coulomb energy

Labels = correlation energy per particle



neutral excitation: $L = kR$

Evidence from numerics

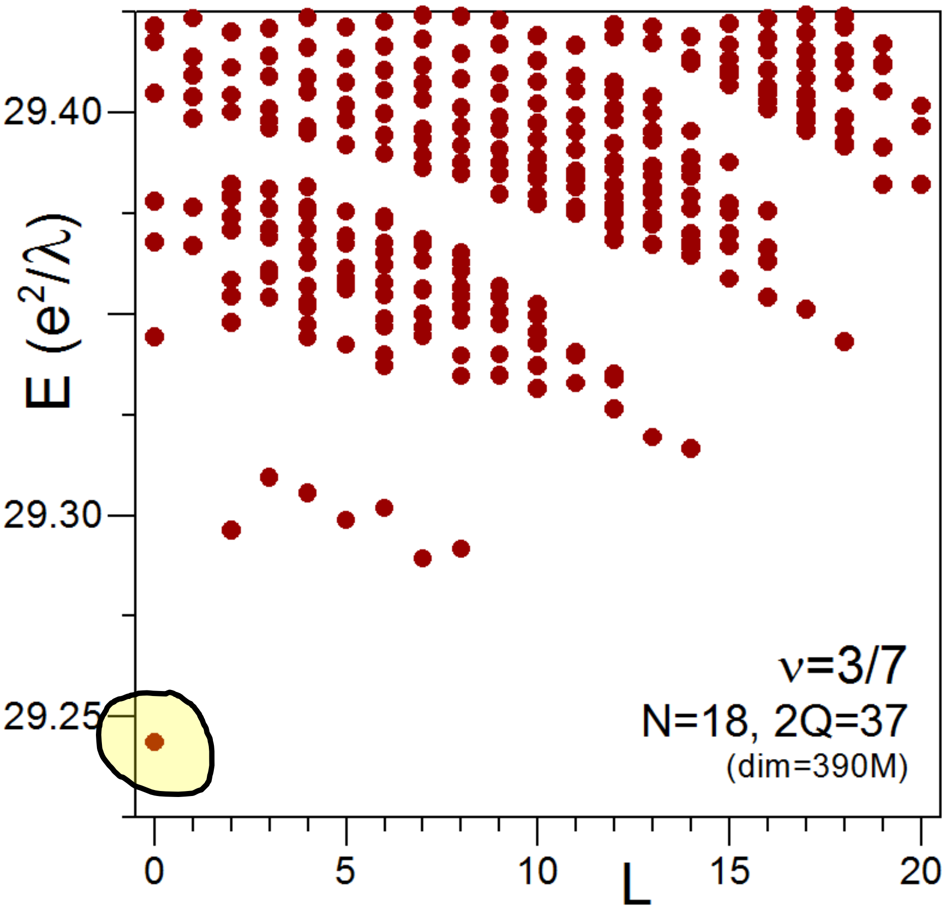


Effective magnetic flux:
2Q* = 2Q – 2(N-1) = 3

CF LLs = ℓ-shells: **ℓ = Q*, Q*+1, …**

Degeneracy = **2ℓ + 1**

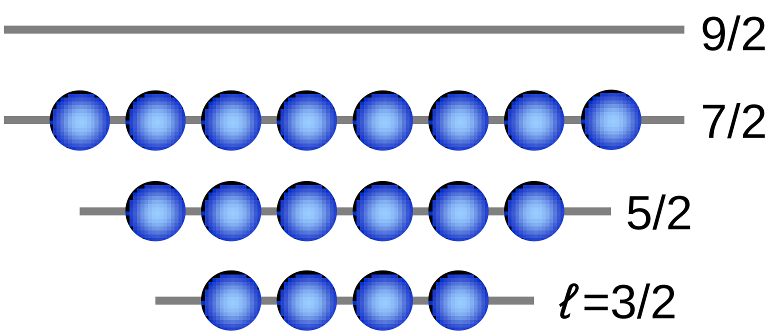
Evidence from numerics



Effective magnetic flux:
$2Q^* = 2Q - 2(N-1) = 3$

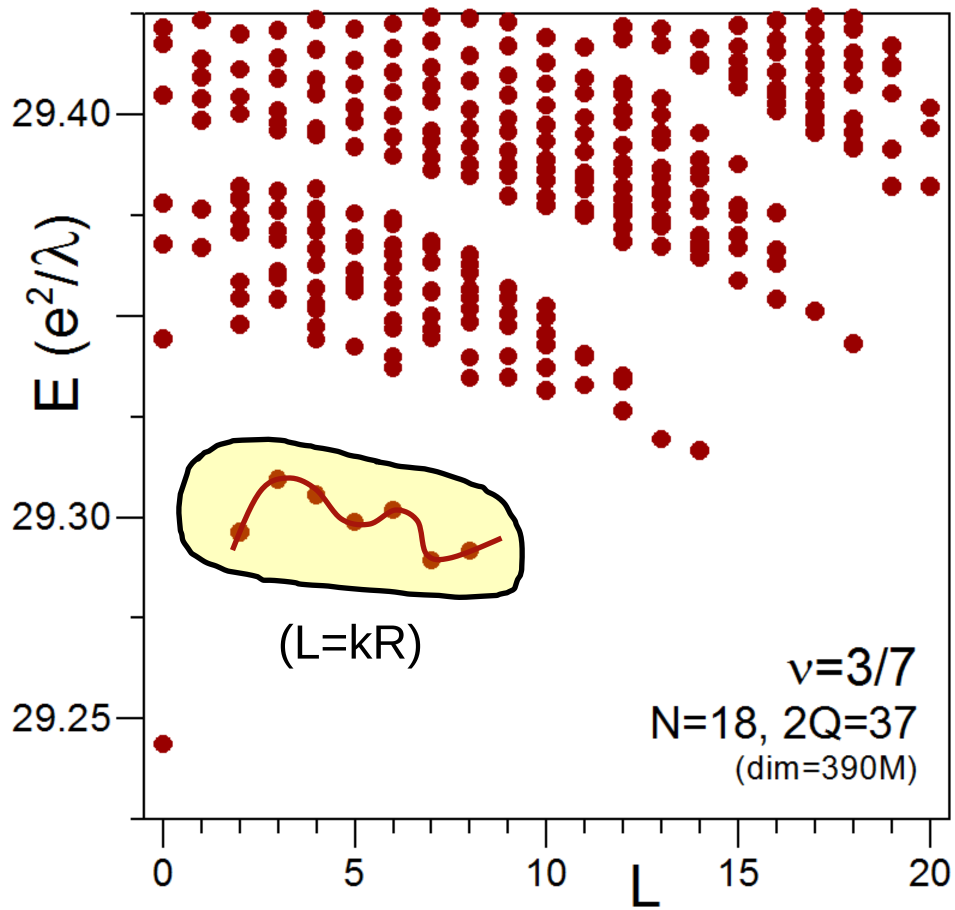
CF LLs = ℓ-shells: **$\ell = Q^*, Q^*+1, \ldots$**

Degeneracy = **$2\ell + 1$**



L=0 (full shells)

Evidence from numerics



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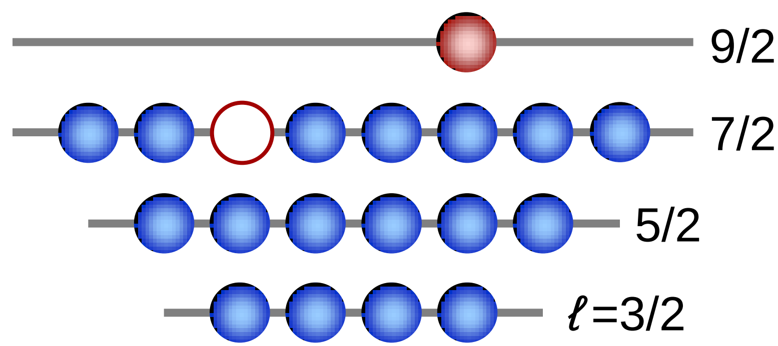
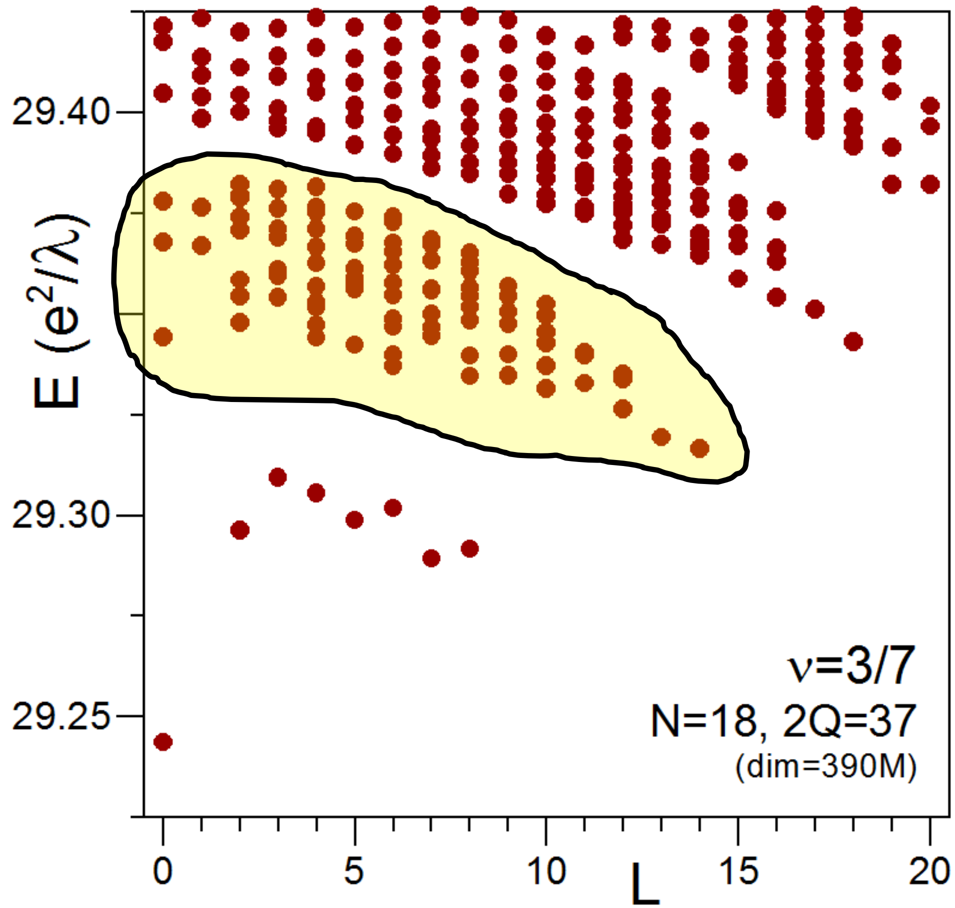


L = 7/2 ⊕ 9/2 ≤ 8 (CF exciton)

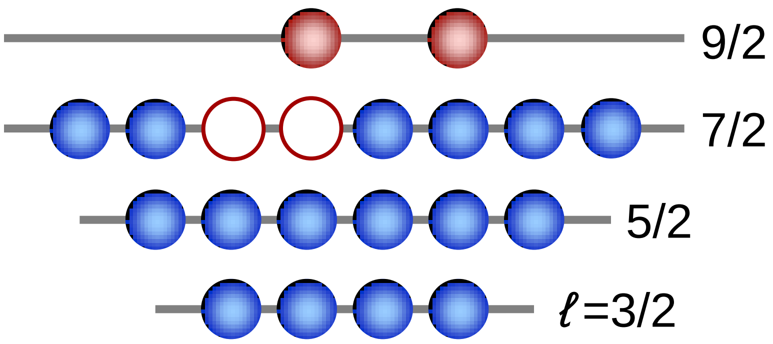
Evidence from numerics



Effective magnetic flux:
2Q* = 2Q – 2(N-1) = 3

CF LLs = ℓ-shells: **ℓ = Q*, Q*+1, …**

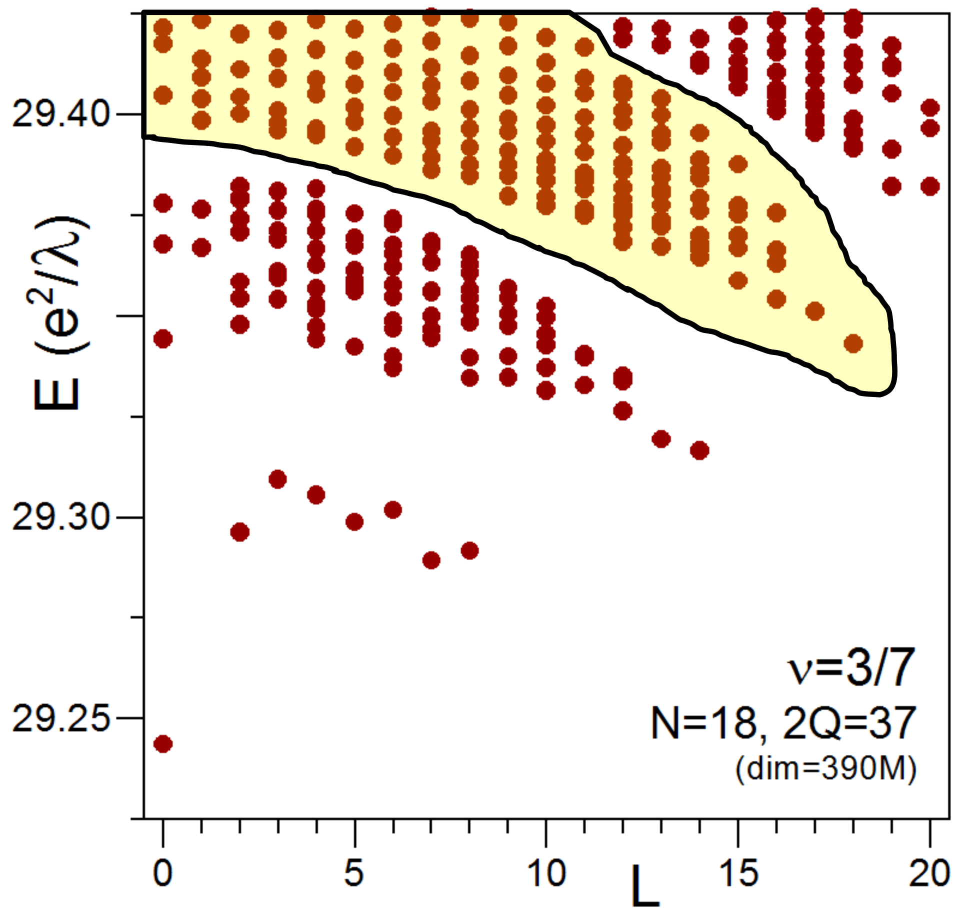
Degeneracy = **2ℓ + 1**



L = $7/2^2 \oplus 9/2^2 \leq 14$ (CF bi-exciton)

(or one exciton with 2 quanta)

Evidence from numerics

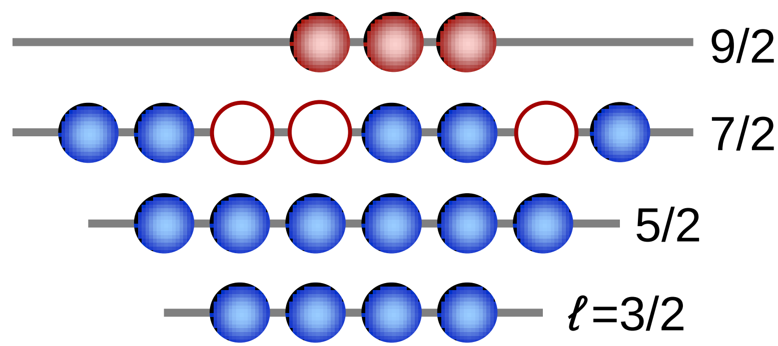


Effective magnetic flux:

$2Q^* = 2Q - 2(N-1) = 3$

CF LLs = ℓ-shells: **$\ell = Q^*, Q^*+1, \dots$**

Degeneracy = **$2\ell + 1$**



$L = 7/2^3 \oplus 9/2^3 \leq 18$ (CF tri-exciton)

(or one or two excitons with 3 quanta)

Configuration interaction

Many-electron Hamiltonian

$$H = \sum_i \frac{1}{2\mu}\left(\mathbf{p}_i - \frac{e}{c}\mathbf{A}_i\right)^2 + \sum_{i<j} V\left(\left|\mathbf{r}_i - \mathbf{r}_j\right|\right)$$

High magnetic field →
- large cyclotron gap
- fractional LL occupation

Model extended 2DEG by $N<\infty$

→ **Haldane spherical geometry**
- 2D symmetry (rotations)
- radial field B
  - (Dirac monopole Φ=2Q)
- LL = shell of ℓ = Q
- LL degeneracy = 2Q+1

Single particle states:
- monopole harmonics $\left|\ell, m\right\rangle$

Interaction:
- e^2/r, chord distance
- (or model repulsions)

CI basis:

***N*-electron determinants**

$$c_1^+ \cdots c_N^+ \left|\mathrm{vac}\right\rangle$$

Hamiltonian (2-body → sparse)

$$H_{2-body} = \sum_{ijkl} V_{ij;kl}\, c_i^+ c_j^+ c_k c_l$$

$$V_{ij;kl} = \sum_m C_{ij}^m C_{kl}^m V(m)$$

Lanczos diagonalization → *E*(*L*)

4. Perspectives/applications

Particles with „memory”

Moore-Read „Pfaffian” wave function
in half-filled Landau level

$$\Psi_{Pf} = \prod_{i<j} (z_i - z_j)^2 \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) e^{\cdots}$$

$$\mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) = \mathcal{A}\left(\frac{1}{z_1 - z_2}\frac{1}{z_3 - z_4}\cdots\right)$$

For a skew-symmetric matrix ***A*** of dim=2*n*
det = (n^th^-degree polynomial in matrix elements)^2 $\equiv$ (Pf)^2

p_x+ip_y superfluid of paired CFs (B*=0, unstable CF Fermi sea)

e/4-charged quasiholes (and quasiparticles)

2^{n-1} degeneracy for 2*n* localized quasiholes → **nonabelian statistics**

Quasiholes cannot be created or destroyed individually/locally

Different states of pinned multiple quasiholes (with different history)
as **qubits** (bits of quantum information) → **quantum computation**

New nonabelian state: ν=3/8

Composite fermion = electron + correlation hole

CFs interact (however weakly) with one another
→ can form quantum liquids (like electrons but not exactly)

Nonabelian „Pfaffian” state of CFs would occur at ν=3/8

Is it really the ground state at this filling?
Experimental evidence for *some* liquid at ν=3/8 (2003)

5. Published results

Possible Anti-Pfaffian Pairing of Composite Fermions at $\nu = 3/8$

Sutirtha Mukherjee,[1] Sudhansu S. Mandal,[1] Arkadiusz Wójs,[2] and Jainendra K. Jain[3]

[1]*Department of Theoretical Physics, Indian Association for the Cultivation of Science, Jadavpur, Kolkata 700 032, India*

[2]*Institute of Physics, Wroclaw University of Technology, 50-370 Wroclaw, Poland*

[3]*Department of Physics, 104 Davey Lab, Pennsylvania State University, University Park, Pennsylvania 16802, USA*

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We predict that an incompressible fractional quantum Hall state is likely to form at $\nu = 3/8$ as a result of a chiral p-wave pairing of fully spin polarized composite fermions carrying four quantized vortices, and that the pairing is of the anti-Pfaffian kind. Experimental ramifications include quasiparticles with non-Abelian braid statistics and upstream neutral edge modes.

DOI: 10.1103/PhysRevLett.109.256801 PACS numbers: 73.43.−f, 05.30.Pr, 71.10.Pm

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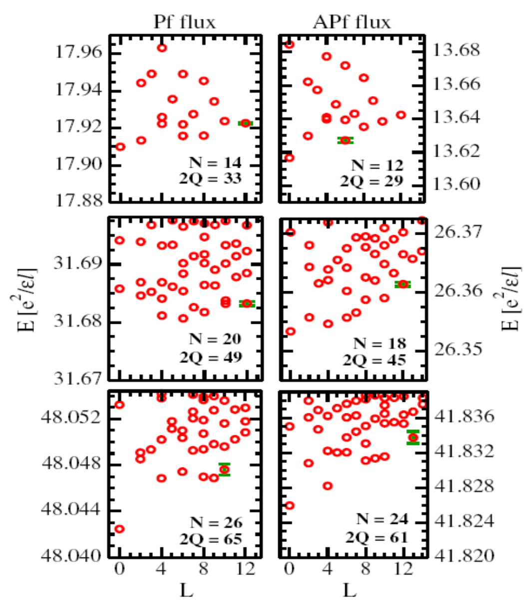

PHYSICAL REVIEW B **85**, 195108 (2012)

Optically induced charge conversion of coexistent free and bound excitonic complexes in two-beam magnetophotoluminescence of two-dimensional quantum structures

J. Jadczak,* L. Bryja, and A. Wójs
Institute of Physics, Wroclaw University of Technology, 50-370 Wroclaw, Poland

M. Potemski
Laboratoire National des Champs Magnétiques Intenses, CNRS-UJF-UPS-INSA, 38042 Grenoble, France

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We report on extensive polarization-resolved photoluminescence (PL) studies of a variety of excitonic complexes formed in high-quality symmetric GaAs quantum wells containing a high-mobility two-dimensional (2D) hole gas in a broad range of magnetic fields from 0 to 23 T and under two-beam illumination, allowing for dynamical control of the hole concentration beyond the point of conversion from *p*- to *n*-type structures. We have demonstrated charge conversion between positive and negative complexes bound to acceptors in the well, differing from the charge conversion of free trions due to charge reflection symmetry breaking by a fixed impurity, leaving a qualitative trace (exchange splitting) in the PL spectrum. The effect of switching between the electron and hole gases (in the same well) on different emission lines has also allowed us to distinguish the (direct and cyclotron-satellite) emission lines from positive trions moving almost freely in the quantum well and bound to nearby ionized acceptors in the barrier, thus demonstrating their coexistence in high-quality structures.

DOI: 10.1103/PhysRevB.85.195108 PACS number(s): 71.35.Ji, 71.35.Pq, 73.21.Fg, 78.20.Ls

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